An Unstructured Mesh Approach to Nonlinear Noise Reduction

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Goal

Motivated by the Air Force Research Lab's (AFRL) Hall Effect Thruster (HET) experimentation, our goal is to denoise signals originating from smooth, chaotic attractors.

Background

Takens' Theorem

Let M be a compact manifold of dimension m. For pairs (φ, y) with $\varphi \in C^2(M, M)$ and $y \in C^2(M, \mathbb{R})$ it is a generic property that the map $\Phi_{(\varphi,y)}(x) := (y(x), y(\varphi(x)), \ldots, y(\varphi^{2m}(x)))$ is an embedding.

With φ, a **time-delay** of τ, and y, an **observable** such as a projection onto one coordinate, we can recover the original manifold from the sequence of time-delayed coordinates. We call these time-delayed coordinates the **shadow manifold**, and we show both a 2-D projection of the Lorenz system manifold and a 2-D shadow manifold of the Lorenz system below.

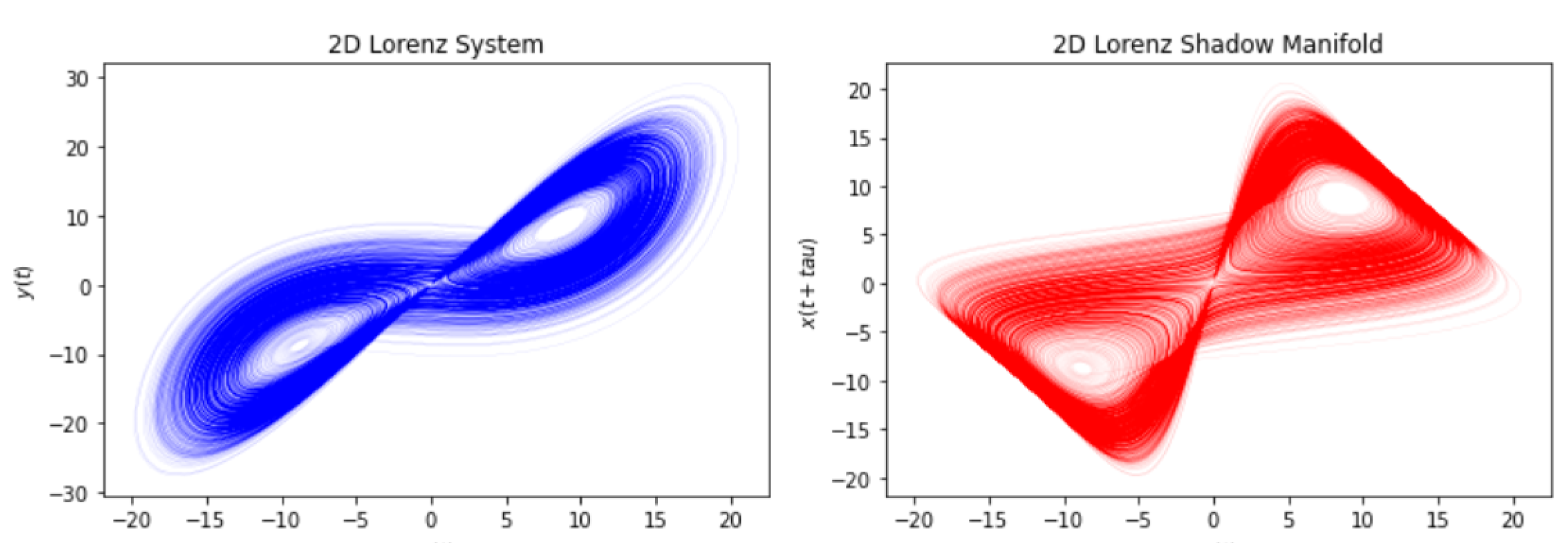


Figure: The 2-D projection of the Lorenz attractor and its X-shadow manifold.

The denoising algorithm involves grouping points based on their locations in the shadow manifold. Previous attempts used a grid-based uniform mesh, resulting in extreme differences between cell density. In our algorithm, we utilize a **Voronoi diagram** instead.

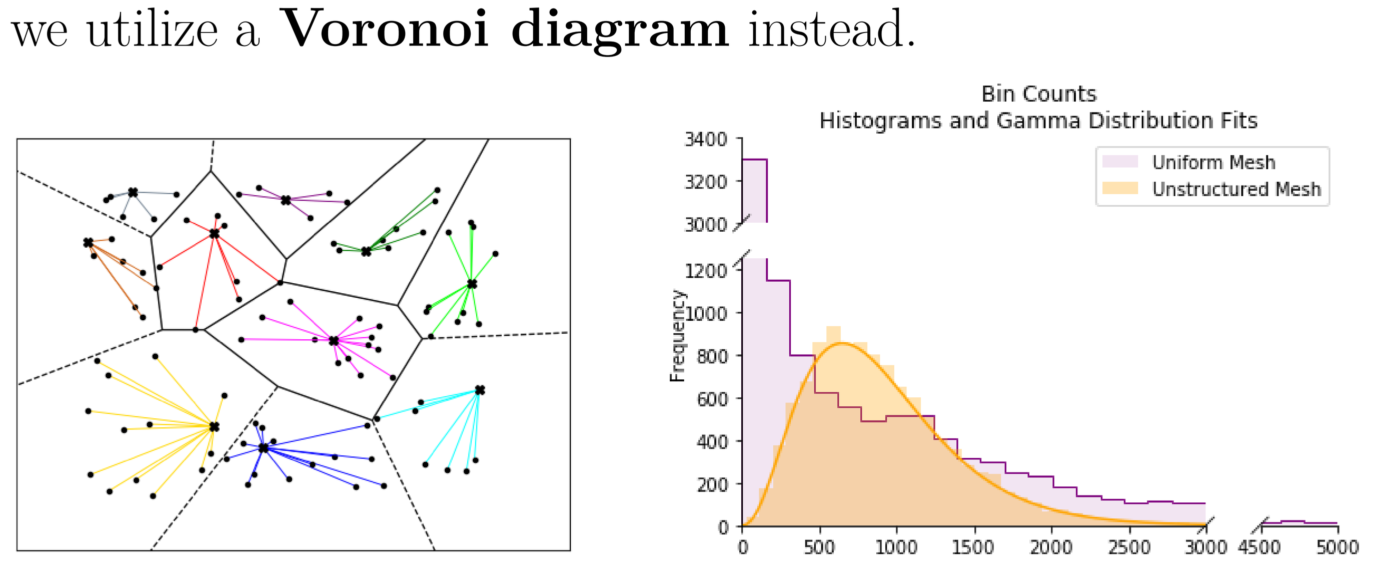


Figure: Using a Voronoi diagram to group points of the manifold vastly lowers cell density variance, as seen in the histogram.

References

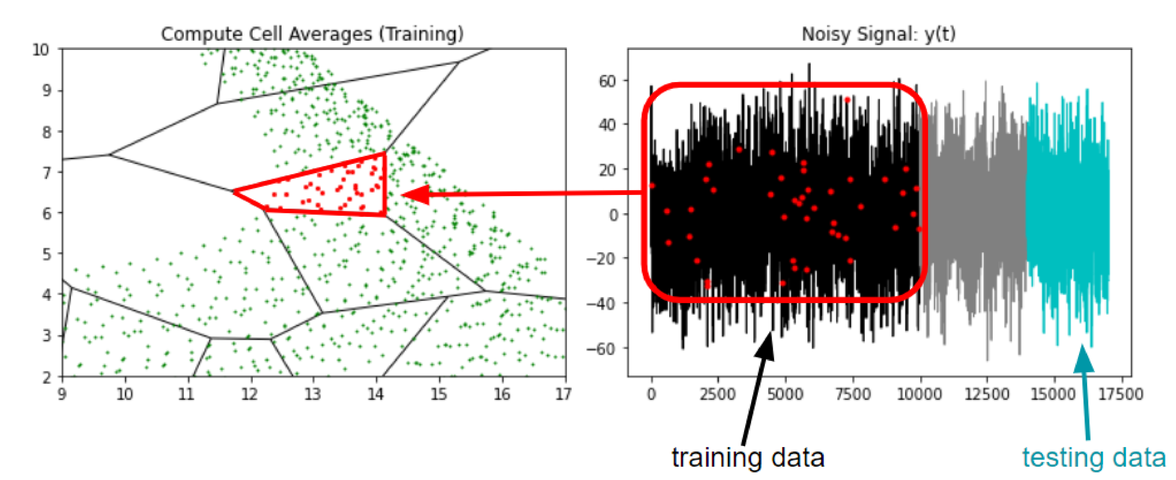
[1] S.J. Araki, J. Koo, R.S. Martin and B. Dankongkakul, "A grid-based nonlinear approach to noise reduction and deconvolution for coupled systems," *Physica D (Submitted)*, 2019.

[2] F. Takens, "Detecting strange attractors in turbulence," in *Dynamical systems and turbulence, Warwick 1980*, pp. 366-381, Springer, 1981.

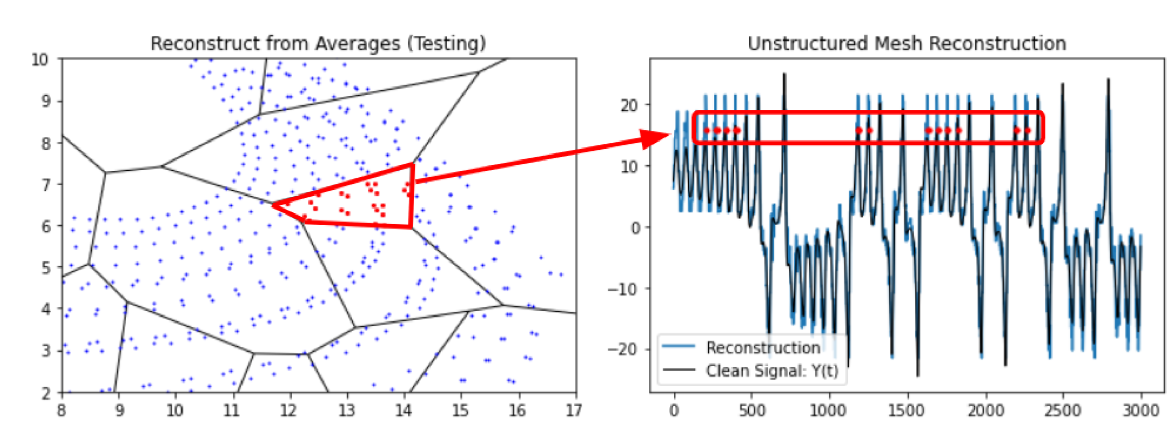
[3] G. Sugihara, R. May, H. Ye, C.-h Hsieh, E. Deyla, M. Fogarty, and S. Munch, "Detecting causality in complex ecosystems," *Science*, vol. 338, no. 6106, pp. 496-500, 2012.

Algorithm

1. Given a clean signal $X(t)$ and a corrupted signal $\widetilde{Y}(t)$ originating from the same chaotic attractor, assign training and testing intervals.
2. Time delay embed the training data of $X(t)$ and select a subset of the embedded data points to construct a Voronoi diagram.
3. Locate the points belonging to the training data of $\widetilde{Y}(t)$ that correspond to the same cell V_i of the Voronoi diagram and compute their average value A_i.



4. Identify the time that each embedded testing point of $X(t)$ belonging to V_i was sampled at and assign in the reconstruction at that time the value A_i.



5. Repeat steps 3 and 4 for all cells of the Voronoi diagram.

Convergence

An optimal number of Voronoi cells is found, and this is used in comparing convergence.

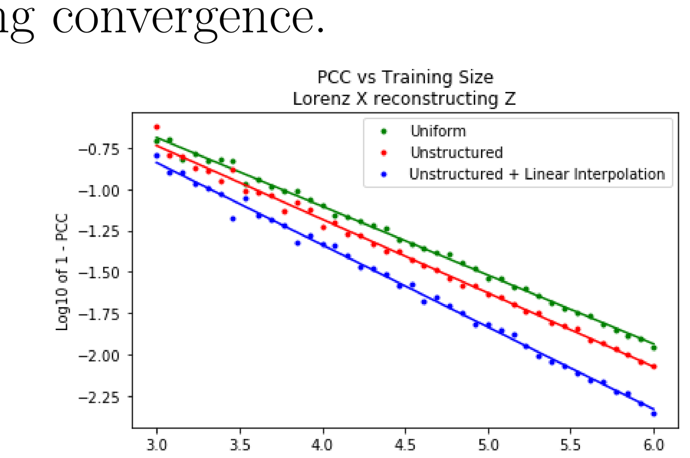


Figure: Convergence comparison between unstructured and uniform meshes.

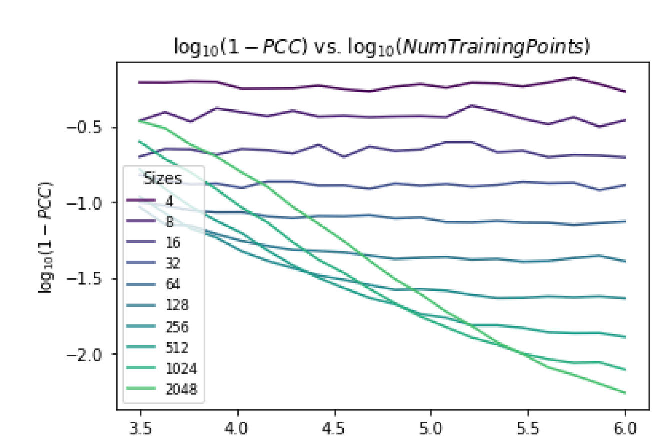


Figure: Convergence comparison between various numbers of Voronoi cells.

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Real-World Application to HET

Given our algorithm's success with synthetic test data, we next applied it to the HET system, from which AFRL sent us several experimentally sampled currents. Taking $X(t)$ to be the provided Anode Cathode signal and $\widetilde{Y}(t)$ to be the Cathode Pearson signal, we continued to produce successful reconstructions.

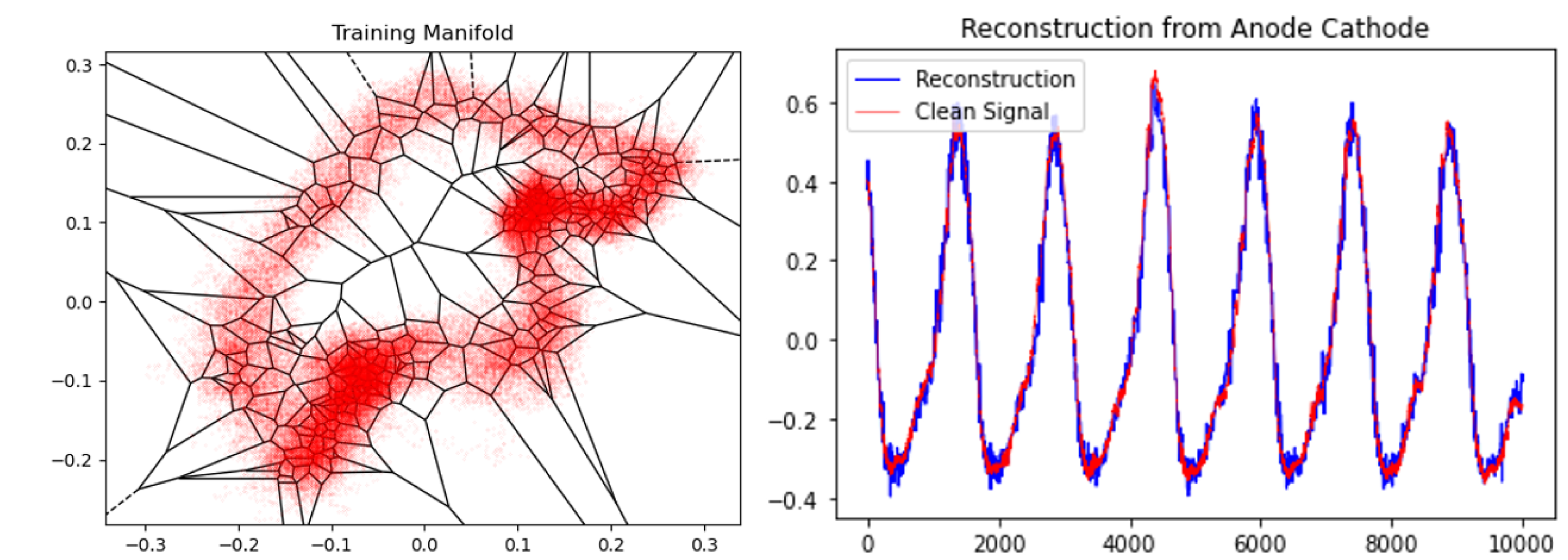

Figure: Training manifold projection and signal reconstruction for HET system

The HET system is naturally higher dimensional than the Lorenz system, so to achieve desirable reconstructions we often conducted time delay embeddings in 8+ dimensions. Though our reconstructions for the HET system did not exhibit true statistical convergence, the trend that we observed was promising, especially given the small amount of available training data.

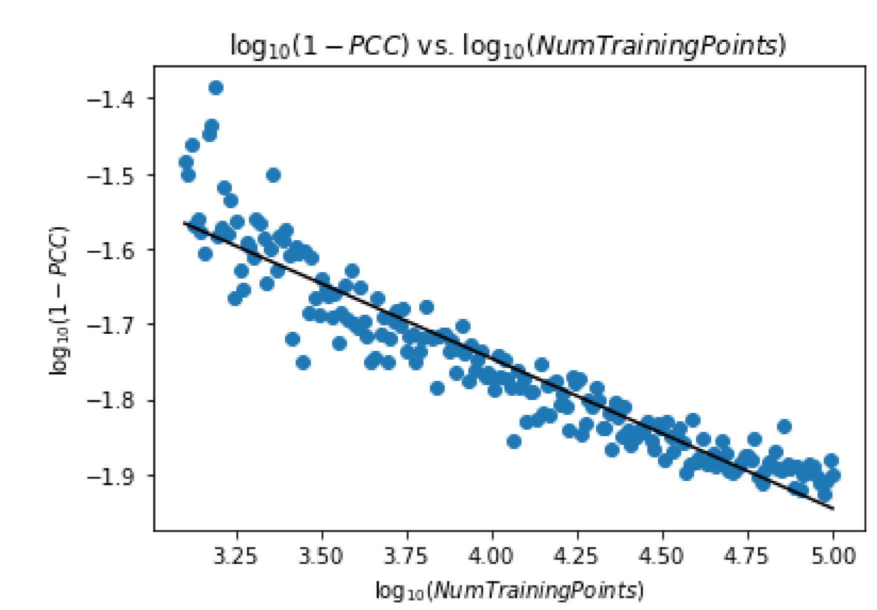

Figure: Convergence of HET reconstructions

Conclusions

In general, our approach was successful and demonstrated statistical convergence as more data was used. By reconstructing known systems such as Lorenz, we were able to test and refine the algorithm, which remained effective when applied to AFRL's HET data.

Future Work

- Apply the algorithm to two noisy signals, instead of one noisy and one clean.
- Use multiple clean signals to recover more information about particularly noisy signals.